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Using origami design principles to fold reprogrammable mechanical metamaterials

Jesse L. Silverberg,1,2 Arthur A. Evans,2 Lauren McLeod,1 Ryan C. Hayward,3 Thomas Hull,1 Christian D. Santangelo,2 Itai Cohen1

Although broadly admired for its aesthetic qualities, the art of origami is now being recognized also as a framework for mechanical metamaterial design. Working with the Miura-ori tessellation, we find that each unit cell of this crease pattern is mechanically bistable, and by switching between states, the compressive modulus of the overall structure can be rationally and reversibly tuned. By virtue of their interactions, these mechanically stable lattice defects also lead to emergent crystallographic structures such as vacancies, dislocations, and grain boundaries. Each of these structures comes from an arrangement of reversible folds, highlighting a connection between mechanical metamaterials and programmable matter. Given origami’s scale-free geometric character, this framework for metamaterial design can be directly transferred to milli-, micro-, and nanometer-size systems.

Metamaterials are rapidly emerging at the frontier of scientific and technological innovation due to their exotic and tunable material properties, which arise from arrangements of smaller units within the bulk system (1–5). Once fabricated, structural modifications are generally difficult, making it rare to find metamaterials that can be reconfigured beyond their original design. Origami-inspired mechanical metamaterials offer enhanced flexibility in performance because their properties are coupled to a dynamically alterable folding pattern (6–10). Hence, multiple stable configurations can manifest from a single structure yielding programmable metamaterials.

We studied the Miura-ori tessellated folding pattern, which has recently been proposed as an origami metamaterial (Fig. 1, A to C, figs. S1 to S3, and movie S1) (14, 15). This historically, this design was invented to efficiently pack solar panels for space missions (17), but the morphology also naturally occurs in leaves (18) and embryonic intestine (19, 20) and generally arises when thin sheets tethered to a surface undergo biaxial compression. The geometry of a Miura-ori is a herringbone pattern that emerges from a series of convex mountain and concave valley creases (Fig. 1C). Vertices are formed when four creases intersect, and four adjacent vertices bound equal-area facets arranged with inversion symmetry (Fig. 1B). This fold pattern defines a lattice characterized by two static crease lengths ℓ1, ℓ2, and one static plane angle α. To quantify folding, a vertex angle ε is required (Fig. 1C); when ε = 180° − 2α, the structure is maximally contracted into a folded state, and when ε = 180°, the structure is an unfolded flat sheet.

If folded from an ideal material with infinite stretching modulus, the Miura-ori would have only one degree of freedom described by ε (14). However, generating samples from laser-cut sheets of paper and mylar (Fig. 1C) (see supplementary materials for details), we find additional degrees of freedom, as evidenced by soft bending modes (14, 15). As an extreme example, the Miura-ori supports highly localized heterogeneity introduced after folding by applying force to a vertex in the normal direction and popping it into a categorically different mechanically stable state (Fig. 2A, fig. S4A, and movie S2). This pop-through defect (PTD) changes lattice topology by suppressing one fold and bending adjacent facets through an angle φ (Fig. 2, B and C). Quantifying lattice distortion by the surface’s Monge patch mean curvature calculated from three-dimensional (3D) digitized scans, we find that displacements of vertices in the xy plane are negligible within a distance of one unit cell; this compares favorably with theoretical predictions that estimate a distortion decay length ∝ 1/2 the unit cell width (Fig. 2D) (see supplementary materials for details) (21, 22). Although highly localized, PTDs are unlike conventional lattice defects because they involve only elastic facet bending and hence are reversibly removable.

To determine the robustness of PTDs to mechanical perturbations, we theoretically model a single unit cell with one mountain and three valley folds in mechanical equilibrium (similar to the paper model in Fig. 2, B and C). To account for experimentally observed facet bending, two additional symmetrically placed creases are included whose folding angles φ are equal and have equilibrium values of 180° (14, 15). We assume a

Fig. 1. Schematic and 3D reconstruction of the Miura-ori with definition of important geometric parameters. (A) The Miura-ori crease pattern on a flat sheet is depicted with alternating mountain (red) and valley (blue) folds. (B) Each facet is bounded by four vertices labeled 1 through 4. (C) An experimentally recorded 3D image of a 4 by 4 laser-cut Miura-ori with two in-plane projections shows the folded structure at equilibrium. The color bar gives height in mm.

References and Notes

1Physics Department, Cornell University, Ithaca, NY 14853, USA. 2Department of Physics, University of Massachusetts, Amherst, MA 01003, USA. 3Department of Polymer Science and Engineering, University of Massachusetts, Amherst, MA 01003, USA. 4Department of Mathematics, Western New England University, Springfield, MA 01119, USA.

*Corresponding author. E-mail: JLS533@cornell.edu

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Nevertheless, such constraints are still malleable; the value of $e$ where $K_y^{\text{defect}}$ increases is shifted simply by varying $n$. Thus, PTDs make attractive elements for metamaterial design because the onset and strength of $K_y^{\text{defect}}$ are rationally tunable, and moreover, these “mechanical pixels” can be activated on demand to vary the bulk compressive properties. Although PTDs were manually introduced here, soft robotics-based technology (25), microelectromechanical systems actuator arrays (26), or even thermal fluctuations (27), provide alternative avenues to introduce and remove PTDs.

To understand both the density dependence and compression dependence of the Miura-ori with a PTD, we analytically calculated the intrinsic PTD compressive modulus, $K_y^{\text{PTD}}$ (see supplementary materials for details). This was found by noting that the stress a PTD exerts is proportional to its extension relative to the neighboring undefected unit cells. At low defect densities for a lattice of arbitrary size, the compressive modulus of a lattice with a PTD can be expressed in a dipole expansion, $K_y^{\text{PTD}} = K_y^{\text{normal}} + nK_y$, where $K_y$...
is a function of $c$ averaged over the nondefected unit cells. This expression is valid when $n << 1$ and holds for a lattice with multiple defects as long as defect interactions can be neglected. Moreover, it suggests a data collapse with a one-parameter fit to the theory that sets the scale of $K_p$. Indeed, we find such a collapse (Fig. 2F, inset) and excellent agreement with the analytic expression (Fig. 2F, inset, black line), demonstrating that the contribution of PTDs to the modulus at low densities is linearly additive. This mechanical response is useful for design purposes because it offers great simplicity in predicting the compressive modulus of a Miura-ori with PTDs.

With increasing density, PTD interactions become important, particularly when pairs of defects are on adjacent vertices. There are four unique defect pairs: 1–2, 1–3, 1–4, and 2–4, with the other combinations degenerate by symmetry (Fig. 1B and Fig. 3, A to D). Folding 4 by 4 lattices the other combinations degenerate by symmetry and finally reduces to a nearly defect-free value at

$$\text{ulus first increases during compression similar to}$$

moduli diverge as the compression increases (Fig. 3F, red and blue dots). During compression, facets near the defect sites are increasingly bent as $c$ decreases, leading to the enhanced stiffness. This enhancement progresses until defect site 4 folds under defect site 2 and all four creases on the facet flip. The defect sites then collapse onto, and overlap with, nearby vertices (Fig. 3F) so that the center lattice site is tuck under near the adjacent cell (movie S4). Visually inspecting the 2–4 defect pair reveals a notable consequence: This defect configuration generates a lattice vacancy analogous to those seen in crystallographic systems (28).

To understand why the 2–4 defect pair is special, we compute the mean curvature from 3D scans of laser-cut sheets (Fig. 3, A to D). Comparing the four cases, we see that the 2–4 pair is distinguished by overlapping bending facets of equal magnitude but opposite sign, as quantified by the zero mean curvature on the central facet. This cancellation is possible due to the symmetry of the 2–4 placement and the reversibility of a PTD’s deformation of a Miura-ori lattice. When fully compressed, the defect-modified crease pattern is compatible with Maekawa’s theorem for flat-foldability (Fig. 3G and fig. S8), and hence the compressive modulus is restored to a nearly defect-free value.

Recognizing the 2–4 defect pair as a lattice vacancy immediately suggests that the Miura-ori supports other types of crystallographic defects (23). For example, a column of vacancies forms an edge dislocation (Fig. 4A), and several adjacent edge dislocations form a grain boundary (Fig. 4B). In both cases, these reconfigurations of the lattice are flat-foldable; however, non-flat-foldable configurations are possible as well. For example, a column of PTDs on alternating vertices generates a hinge-like structure that allows the Miura-ori to easily bend out of plane (Fig. 4C). Conversely, a column of PTDs on consecutive vertices forms a corrugated structure that is highly resistive to out-of-plane bending (Fig. 4D). In fact, rationally introducing patterns of defects as design elements swiftly generates a vast library of origami-inspired mechanical metamaterials. To demonstrate in experiments the ability to place these features without permanently altering the crease pattern, we programmed eight configurations into a single sheet with 13 consecutive reprogramming events (Fig. 4E) (29). Measuring $K_p$ for a lattice compressed to 68% of its initial width, we found that once calibrated (dashed lines), we are able to predict and realize target modulus values when defect concentration and interactions are altered (solid black lines). This capability to dynamically reprogram elastic properties illustrates the power of our approach.

Extending programmable metamaterial design principles to self-folding robotic (25, 30–33) and polymer (8–10, 13, 16) systems opens the door to engineering devices that can alter their mechanical functionality on demand. Such enhanced capabilities would move these transforming systems whose function depends on configuration from what was merely science fiction in past decades to real-world applications.

REFERENCES AND NOTES

EARLY SOLAR SYSTEM

Stellar origin of the $^{182}$Hf cosmochronometer and the presolar history of solar system matter

Maria Lugaro,1 Alexander Hegel,2,3 Dean Osrin,1 Stephane Goriely,4 Kai Zuber,5 Amanda I. Karakas,6,7 Brad K. Gibson,8,9,10 Carolyn L. Doherty,1
John C. Lattanzio,1 Ulrich Ott1

Among the short-lived radioactive nuclei inferred to be present in the early solar system via meteoritic analyses, there are several heavier than iron whose stellar origin has been poorly understood. In particular, the abundances inferred for $^{182}$Hf (half-life = 8.9 million years) and $^{129}$I (half-life = 15.7 million years) are in disagreement with each other if both nuclei are produced by the rapid neutron-capture process. Here, we demonstrate that contrary to previous assumption, the slow neutron-capture process in asymptotic giant branch stars produces $^{182}$Hf. This has allowed us to date the last rapid and slow neutron-capture events that contaminated the solar system material at ~100 million years and ~30 million years, respectively, before the formation of the Sun.

Radioactivity is a powerful clock for the measurement of cosmic times. It has provided us the age of Earth ($\tau$), the ages of old stars in the halo of our Galaxy ($\tau$), the age of the solar system ($\tau$), and a detailed chronometry of planetary growth in the early solar system ($\tau$). The exploitation of radioactivity to measure time scales related to the presolar history of the solar system material, however, so far has been hindered by our poor knowledge of how radioactive nuclei are produced by stars. Of particular interest are three radioactive isotopes heavier than iron: $^{107}$Pd, $^{129}$I, and $^{182}$Hf, with half-lives of 6.5 million years (My), 15.7 My, and 8.9 My, respectively, and initial abundances (relative to a stable isotope of the same element) in the early solar system of $^{107}$Pd/$^{108}$Pd = 5.9 $\times$ 10$^{-5}$ (6), $^{129}$I/$^{131}$I = 1.19 $\pm$ 0.20 $\times$ 10$^{-4}$ (7), and $^{182}$Hf/$^{180}$Hf = 9.72 $\pm$ 0.44 $\times$ 10$^{-6}$ (8). The current paradigm is that $^{129}$I and $^{182}$Hf are mostly produced by rapid neutron captures (the $\tau$ process), in which the neutron density is relatively high ($\sim$10$^{29}$ cm$^{-3}$), resulting in much shorter time scales for neutron capture than for $\beta$-decay (9). The $\tau$ process is believed to occur in neutron star mergers or peculiar supernova environments (10, 11). In addition to the $\tau$ process, $^{107}$Pd is also produced by slow neutron captures (the $\alpha$ process), in which the neutron density is relatively low ($<$$10^{23}$ cm$^{-3}$), resulting in shorter time scales for $\beta$-decay than for neutron capture, the details depending on the $\beta$-decay rate of each unstable isotope and the local neutron density ($\tau$). The main site of production of the $s$ process elements from Sr to Pb in the Galaxy is in asymptotic giant branch (AGB) stars (12), the final evolutionary phase of stars with initial mass lower than ~10 solar masses ($M_{\odot}$). Models of the $s$ process in AGB stars have predicted marginal production of $^{182}$Hf (13) because the $\beta$-decay rate of the unstable isotope $^{182}$Hf at stellar temperatures was estimated to be much faster (14) than the rate of neutron capture leading to the production of $^{182}$Hf (Fig. 1).

Uniform production of $^{182}$Hf and $^{129}$I by the $\tau$ process in the Galaxy, however, cannot self-consistently explain their meteoritic abundances (15–17). The simplest equation for uniform production (UP) of the abundance of a radioactive isotope in the Galaxy, relative to a stable isotope of the same element produced by the same process, is given by

\[
\frac{N_{\text{radio}}}{N_{\text{stable}}} = \frac{P_{\text{radio}}}{P_{\text{stable}}} \times \frac{\tau}{T} \tag{1}
\]

where $N_{\text{radio}}$ and $N_{\text{stable}}$ are the abundances of the radioactive and stable isotopes, respectively; $P_{\text{radio}}/P_{\text{stable}}$ is the ratio of their stellar production rates; $\tau$ is the mean lifetime of the radioactive isotope; and $T \sim$ 10$^{9}$ years is the time scale of the evolution of the Galaxy. Some time during its presolar history, the solar system material became isolated from the interstellar medium characterized by UP abundance ratios. Assuming that both $^{129}$I and $^{182}$Hf are primarily produced by the $\tau$ process, one obtains inconsistent isolation times using $^{129}$I/$^{131}$I or $^{182}$Hf/$^{180}$Hf: 72 My or 15 My, respectively, before the solar system formation (17). This conundrum led Wasserburg et al. (15) to hypothesize the existence of two types of $\tau$ process events. Another proposed solution is that the $^{107}$Pd, $^{129}$I, and $^{182}$Hf present in the early solar system were produced by the neutron burst that occurs during core-collapse supernovae (18–20). This does not result in elemental production, but the relative isotopic abundances of each element are strongly modified because of relatively high neutron densities with values between those of the $s$ and $\tau$ processes.
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Editor's Summary

Folding robots and metamaterials

The same principles used to make origami art can make self-assembling robots and tunable metamaterials—artificial materials engineered to have properties that may not be found in nature (see the Perspective by You). Felton et al. made complex self-folding robots from flat templates. Such robots could potentially be sent through a collapsed building or tunnels and then assemble themselves autonomously into their final functional form. Silverberg et al. created a mechanical metamaterial that was folded into a tessellated pattern of unit cells. These cells reversibly switched between soft and stiff states, causing large, controllable changes to the way the material responded to being squashed.

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